Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2019

Question/Answer booklet

MATHEMATICS SPECIALIST

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Section Two: Calculator-assumed

WA student number: In figures

In words

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Ref: 19-052

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	9	9	50	47	35
Section Two: Calculator-assumed	10	10	100	85	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section Two: Calculator-assumed **65% (85 Marks)**

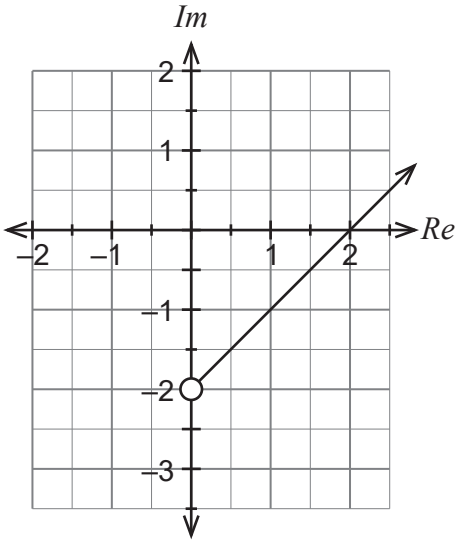
This section has **10** questions. Answer all questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 10 **(5 marks)**

The sketch of the locus of a complex number $z = x + iy$ is shown below.

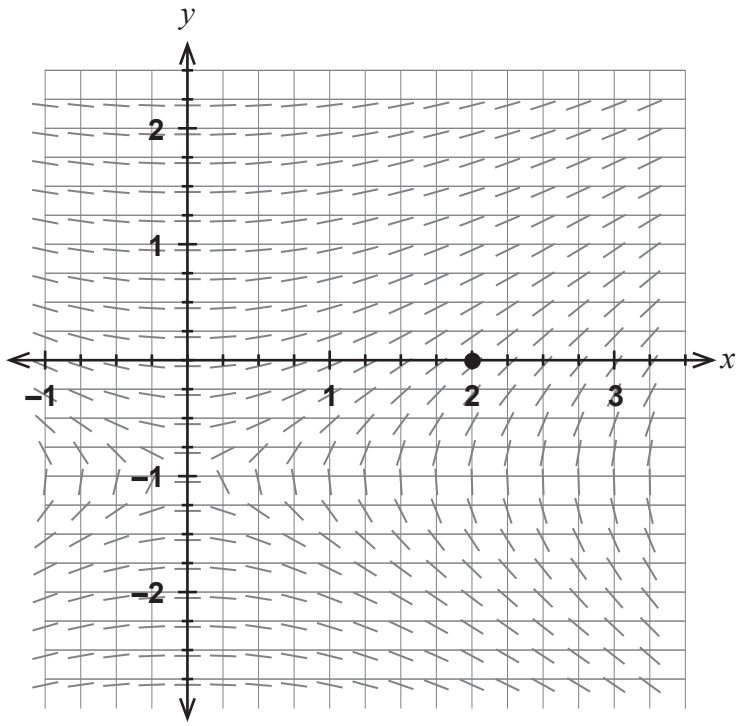


(a) Given that the equation for the above locus is written as $Arg(z - z_0) = k\pi$, determine the value of the constants z_0 and k. (2 marks)

(b) Determine the minimum value for $|z - i|$ as an exact value. (3 marks)

Question 11 **(6 marks)**

The slope field given by $\frac{dy}{dx} = \frac{x}{2y+2}$ is shown in the diagram below.



(a) Calculate the value of the slope field at the point $(2,0)$. (1 mark)

(b) On the diagram above, draw the solution curve that contains the point $(2,0)$. (2 marks)

See next page

(c) Determine the equation for the solution curve that contains the point $(2,0)$. (3 marks)

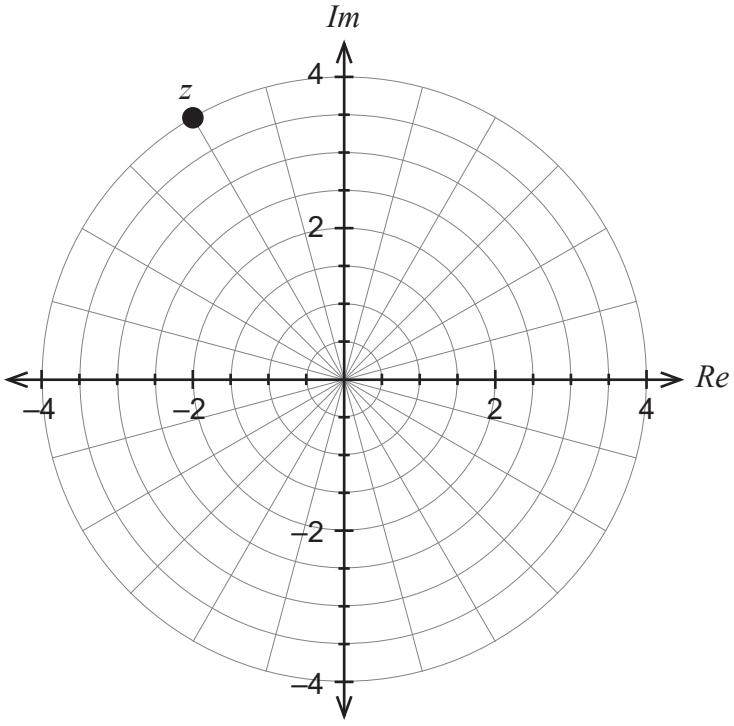
See next page

Question 12 **(10 marks)**

Let $w = \dfrac{1-i}{2\sqrt{2}}$.

(a) Express w in the form $w = r\,cis\theta$, where $-\pi < \theta \leq \pi$. (2 marks)

The complex number z is represented in the Argand diagram below.



(b) Express z exactly in the form $z = a + bi$. (2 marks)

See next page

(c) Determine the exact polar form for wz and w^2z. (2 marks)

(d) On the Argand diagram on page 6, plot the position for wz and w^2z. Ensure that each position is labelled clearly. (2 marks)

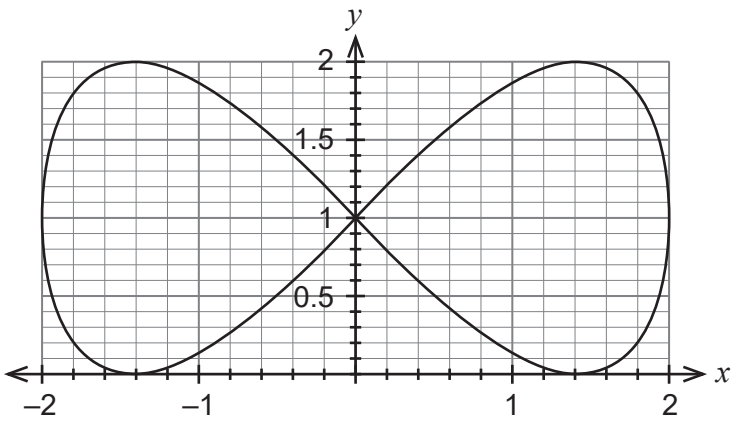
Consider the geometric transformation(s) applied to transform $z \rightarrow wz \rightarrow w^2z \rightarrow w^3z$ etc.

(e) Describe the geometric transformation(s) performed by the successive multiplication by w. (2 marks)

See next page

Question 13 **(10 marks)**

The path of a particle is shown below. This particle moves so that its position vector $\underset{\sim}{r}(t)$ is given by $\underset{\sim}{r}(t) = \begin{bmatrix} -2\cos\left(\frac{t}{2}\right) \\ 1 - \sin(t) \end{bmatrix}$ metres, where t is the number of seconds the particle has been in motion.



(a) Determine the starting position of the particle and mark this as point A on the diagram above. (1 mark)

(b) Determine the initial velocity of the particle and illustrate this on the diagram above. (3 marks)

See next page

(c) Write the expression, in terms of trigonometric functions, for the distance the particle would travel in completing one circuit of the given path. Do **not** evaluate this expression. (3 marks)

(d) Determine the Cartesian equation for the path of the particle. (3 marks)

Question 14 **(10 marks)**

Trucks carrying iron ore for the Croc Rock mining company arrive at a weighing station. The service time T per truck is defined to be the time elapsed from the moment a truck enters the station zone, including the time to be positioned and then weighed, up to the time it leaves the zone.

It is known that the population mean $\mu(T) = 80$ seconds and the population standard deviation $\sigma(T) = 20$ seconds.

At the Croc Rock weighing station, 100 trucks are weighed.

(a) State the (approximate) distribution of the sample mean service time per truck for the 100 trucks. (3 marks)

(b) What is the probability that the sample mean service time will be more than 83 seconds? (2 marks)

Suppose that more than 100 trucks were weighed at the Croc Rock weighing station.

(c) How would this affect your answer to part (b)? Explain without recalculation. (2 marks)

See next page

It is desired that the probability that the sample mean service time will be between 80 seconds and 82 seconds is greater than 40%.

(d) Determine the minimum number of trucks that will need to be weighed. (3 marks)

See next page

Question 15 **(9 marks)**

A random sample of n commuters in Melbourne in August 2018 found that the average time to commute to work was 40 minutes. Repeated sampling of the mean indicated that the standard deviation of the sample mean was 3 minutes.

(a) Determine a 90% confidence interval for the population mean commuting time μ to work, correct to 0.01 minutes. (3 marks)

Another random sample of $2n$ commuters in November 2018 found that the average time to commute to work was 45 minutes. Assume that both the August and November samples were drawn from the same population.

(b) What is the standard deviation of the sample mean for the November sample, correct to 0.01 minutes? (2 marks)

See next page

Suppose that the August and November samples are combined to form a sample with $3n$ commuters. Consider 90% confidence intervals for the following samples for the purpose of determining the population mean commuting time μ.

90% confidence interval	Sample	Size
A	August	n
N	November	$2n$
C	Combined	$3n$

(c) Which of the three confidence intervals, A, N or C, will provide the greatest precision in determining the population mean μ? Justify your answer. (2 marks)

(d) Which of the three confidence intervals, A, N or C, contains the true value of the population mean μ? Justify your answer. (2 marks)

See next page

Question 16 **(12 marks)**

Plane Π_1 has Cartesian equation $z = 2x + y + 4$.

(a) Determine a vector that is normal to plane Π_1. (2 marks)

Line L has equation $\underset{\sim}{r} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

(b) Determine the point of intersection between line L and plane Π_1. (3 marks)

See next page

Plane Π_2 contains line L and is perpendicular to plane Π_1.

(c) Determine the vector equation for plane Π_2. (4 marks)

Sphere S has vector equation $|\underset{\sim}{r} - (3\underset{\sim}{i} + \underset{\sim}{j} + 4\underset{\sim}{k})| = \sqrt{35}$.

(d) Determine whether line L is a tangent to sphere S. Justify your answer. (3 marks)

See next page

Question 17 **(8 marks)**

In Australia, the killing of humpback whales was banned in 1963.

At the end of 2018, 45 years later, the population P of migrating humpback whales off the coast of Western Australia was estimated at 30 000, i.e. $P(45) = 30\ 000$.

(a) Assuming that the population of humpback whales had been increasing at an instantaneous rate equal to 10% of the population, estimate the number of humpback whales at the end of 1963. (3 marks)

To model the growth in the population from the end of 2018, a marine biologist suggests that the rate of growth be modelled by the equation below.

$$\frac{dP}{dt} = 0.1P - \frac{P^2}{700\ 000}$$

The biologist re-defines $P(0) = 30\ 000$, i.e. t = number of years from the end of 2018.

(b) If $P(t)$ is written in the form $P(t) = \frac{a}{1 + be^{-ct}}$, determine the values of the constants a, b and c. (2 marks)

(c) Hence determine the year during which the population of humpback whales off the coast of Western Australia will reach double that estimated at the end of 2018. (2 marks)

(d) State the major difference in the variation in the population $P(t)$ using the model in part (b) compared with that in part (a). (1 mark)

See next page

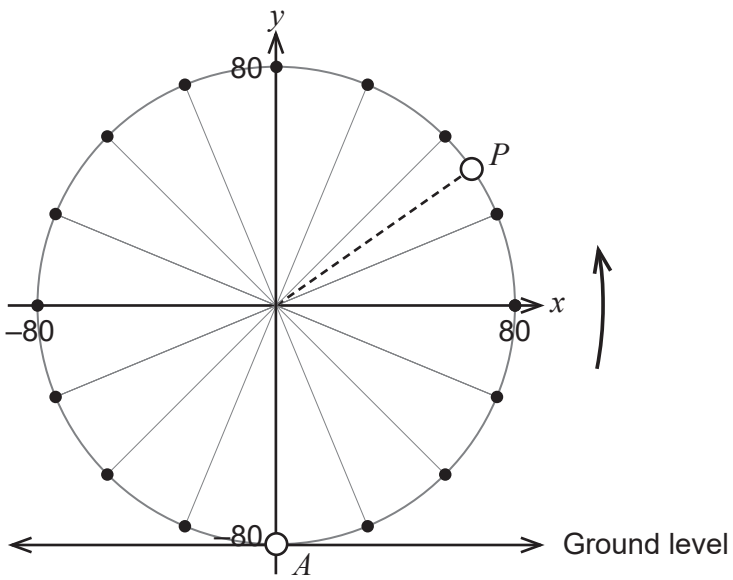
Question 18 **(11 marks)**

A ferris wheel has a radius of 80 metres and rotates in an anticlockwise direction at a rate of one revolution every 72 seconds. The ferris wheel has 16 cars that are equally spaced around the wheel as shown in the diagram.

A coordinate system is set up so that the centre of the ferris wheel is at the origin and the ground level has equation $y = -80$. Passengers begin their ride when a car is at position $A\ (0,-80)$.

Consider a passenger in a car at position P.

Let t = the number of seconds the ride has been in progress from position A.
θ = the angle in radians that the car has rotated from position A.
y = the height of a car above the centre of the ferris wheel (metres).



(a) Show that $\frac{d\theta}{dt} = \frac{\pi}{36}$ radians per second. (1 mark)

(b) Given that $y(\theta) = 80\sin(\theta + \alpha)$, explain why $\alpha = -\frac{\pi}{2}$. (1 mark)

See next page

(c) Determine how quickly a passenger is moving upward when they are 100 metres above the ground, correct to the nearest 0.01 metres per second. (4 marks)

(d) Show that function $y(t)$ satisfies the condition for simple harmonic motion. (2 marks)

A different passenger happens to be in a car that is two cars ahead of a particular car on the ferris wheel.

(e) At what speed, correct to the nearest 0.01 metres per second, is the trailing passenger moving upward when the other passenger is moving downward at exactly the same speed? (3 marks)

See next page

Question 19 **(4 marks)**

Two parallel planes Π_1 and Π_2 have their equations given by:

$\Pi_1 \quad \underset{\sim}{r} \bullet \underset{\sim}{n} = 11$

$\Pi_2 \quad \underset{\sim}{r} \bullet \underset{\sim}{n} = -4 \quad$ where $\underset{\sim}{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

It is known that (2,3,–7) is a point on plane Π_1.

Prove the distance d between the point (2,3,–7) and plane Π_2 is given by $d = \dfrac{15}{\sqrt{a^2 + b^2 + c^2}}$.

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

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